AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
Department of Physics and Engineering Mathematics

Freshman **Survey Paper**

Spring 2020	Course Code: *PHM 013s*	*Deadline: 30 May@4pm*

Mathematics (2)

تعليمات هامة

- غير مسموح بالتشابه بين الأبحاث المقدمة من الطلاب.
- ضرورة مراعاة الأمانة العلمية والدقة فى عرض المراجع التى تم الاستعانة بها
- ضرورة الالتزام بالقواعد المنصوص عليها المرتبطة بشكل وعدد الصفحات المنصوص عليها
- يتم تجميع ملفات الاجابة فى ملف واحد فى صورة PDF ويتم رفعه على صفحة المادة المخصصة على منصة LMS
- ضرورة الالتزام بحجم الملفات المطلوب رفعها وفقا لما هو منصوص عليه بمنصة LMS.
- ضرورة مراعاة تقديم ملف منفصل لكل طالب ومراعاة الالتزام بالقواعد المنصوص عليها المرتبطة بالبيانات والترتيب

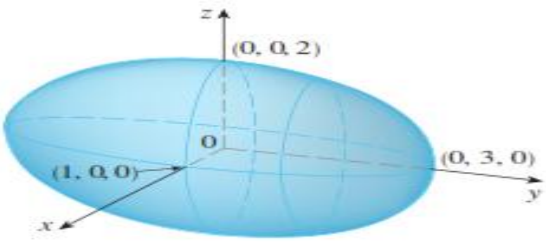
Part 1(70%):Functions Of Several Variables and Multiple Integrals

Write in Three Topics with maximum three pages for each Topic.
(20 % for each Topic and 10% for good arrangement and neat sketches)

1) i) Discuss different types in details of Quadratic Surfaces given by the equation:

$$A\ x^2 + B\ y^2 + C\ z^2 + D\,z = E \qquad (8\ \%)$$

ii) For the Ellipsoid shown in Figure and given by the equation: $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$



Graph the traces of the ellipsoid (You may use Excel Sheets) when (6 %)

(1) $z = K_z$; $K_z = 0 , 1 , 2$ **(2)** $y = K_y$; $K_y = 0 , 2 , 3$ **(3)** $x = K_x$; $K_x = 0 , 1$

iii) Find and sketch the domain of the following function then find the range

$$f\,(x , y , z) = \ln\,(\,16 - 4\,x^2 - 4\,y^2 - z^2\,) \qquad (6\ \%)$$

2) i) Discuss the Partial Derivatives and Chain Rules in different two cases and explain the use of Partial Differentiation in obtaining critical points of the Surfaces with applications. (8 %)

ii) Suppose f is differentiable function of x and y and
$g\,(u , v) = f\,(e^u + \sin v\,, e^u + \cos v\,)$
Use the following table of values to evaluate $g_u\,(0 , 0)$ **and** $g_v\,(0 , 0)$. **(6 %)**

	f	g	f_x	f_y
(0,0)	6	3	8	4
(1,2)	2	5	3	6

**iii) Find and classify the critical point (if any) of the Surface: $f(x, y) = y^2 - x^2$
Check the critical point (x^*, y^*) grahically (You may use Excel Sheets) by graphing: $f(x, y^*)$; $f(x^*, y)$; $f_x(x, y^*)$; $f_y(x^*, y)$ (6 %)**

3) i) Discuss the Double Integrals in different coordinate systems; Rectangular and Polar with applications. (8 %)

ii) Evaluate $\iint_R \frac{x}{x^2 + y^2}\, dA$, where R is the region inside $x^2 + y^2 = 2y$ and outside $x^2 + y^2 = 1$ (6 %)

**iii) Use Double Integration to Evaluate the Volume (V) bounded by the Cylinder: $x^2 + y^2 = 4$ and the Plane P passing through the points $(6, 0, 0)$; $(0, 6, 0)$ and $(0, 0, c)$ where $0 \le c \le 5$ above xy – plane in terms of c.
Then, Sketch the graph between the angle θ between the plane P and the xy – plane and the Volume (V) when $c = 0, 1, 2, 3, 4, 5$
(You may use Excel Sheets). (6 %)**

4) i) Discuss the Triple Integrals in different coordinate systems; Rectangular ; Cylindrical and Spherical coordinates with applications. (8 %)

ii) Find the volume that lies within the sphere: $x^2 + y^2 + z^2 = 4$, and above the cone: $z = \sqrt{x^2 + y^2}$ and above the xy-plane. (6 %)

**iii) Use Triple Integration to Evaluate the Volume (V) bounded by the Cylinder: $x^2 + y^2 = 16$ and the Plane P passing through the points $(10, 0, 0)$; $(0, 10, 0)$ and $(0, 0, c)$ where $0 \le c \le 5$ above xy – plane in terms of c.
Then, Sketch the graph between the angle θ between the plane P and the xy – plane and the Volume (V) when $c = 0, 1, 2, 3, 4, 5$
(You may use Excel Sheets). (6 %)**

Part 2(30%): Systems of Equations and Eigen values and Eigen vectors

Write in <u>One</u> Topic with maximum three pages.

1) **Discuss the solution of Linear System of Equations by Using Rank of Matrix and LU-Decomposition with applications. Then solve the following problem:**

Find an LU-factorization of the matrix $A = \begin{pmatrix} -4 & 5 & -2 \\ -3 & 2 & -1 \\ 1 & 1 & 0 \end{pmatrix}$**. Then use it to solve the system** $A\,x = b$**, where** $b = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$**.**

2) **Discuss the determination of the Eigen values and Eigen vectors for matrix A with applications in determining power of the matrix. Then solve the following problem:**

For the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{pmatrix}$**, find** A^{2020} **.**

$\boxed{\frac{95}{100}}$

RESEARCH & PROJECT SUBMISSIONS

Program:

Course Code: PHM013s

Course Name: Mathematics

Examination Committee

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Spring Semester – 2020

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Signature/Student Name: Raneem Salah Attia.

Date: **30-5-2020**

Submission Contents

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01: Quadratic surfaces
02: Partial derivative and chain rule
03: Double integral
04: Eigen values and Eigen vectors

01 Quadratic Surfaces

1.1 **discuss different types in details of quadratic surfaces given by the equation:** $Ax^2+By^2+Cz^2+Dz=E$

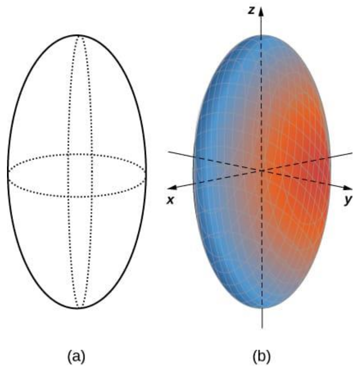
the quadratic surfaces are the dimensional extension of conical parts such as ellipse and parabola as well the excess segments ,and the quadrant sits through general equation ,which is

$$Ax^2+By^2+Cz^2+Dz=E$$

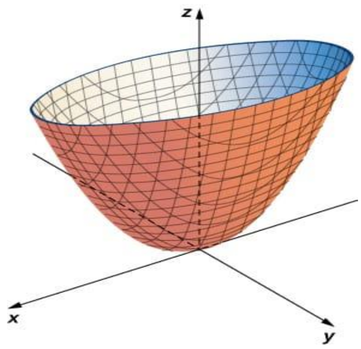
1) Quadratic surface (ellipsoid):

If we suppose that the coefficient of x^2=coefficient of y^2=coefficient of z^2

That's mean A=B=C and if we suppose that the coefficient of Z=0 that's mean D=0 and suppose E=1 we will get ellipsoid shape that's notice that this shape drawn two dimension in form of an ellipse.



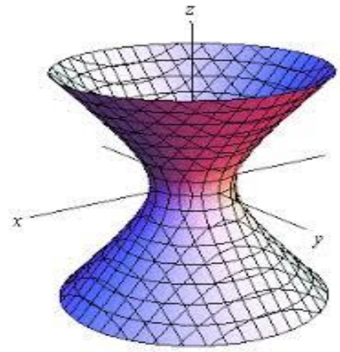
It's his general equation $\longrightarrow$ $\frac{x^2}{A^2}+\frac{y^2}{B^2}+\frac{z^2}{C^2}=1$



2)quadratic surface (parabloid): the ellipse is tracked by the ellipse that exists in each of coordinate levels and yet this should not be the case for all quadruple surfaces as it contains many quadruple surfaces on the other tracks different types of conical sections and is usually referred to as parabloid

its general equation is : $\longrightarrow$ $\frac{x^2}{A^2}+\frac{y^2}{B^2}=\frac{z}{c}$

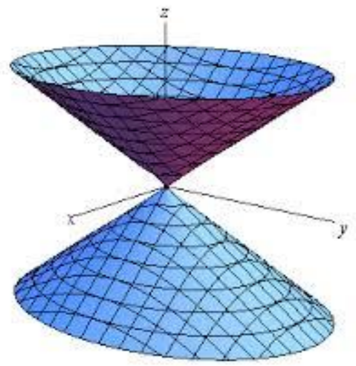
the effect in xy is and ellipse while in xz , yz parabola in addition the parts equivalent of ellipse can have other direction simply by switching between variables to give us a different variable in the linear surfaces as in the two equation : $\frac{x^2}{A^2}+\frac{z^2}{c^2}=\frac{y}{B}$. $\frac{Z^2}{C^2}+\frac{y^2}{B^2}=\frac{x}{A}$ And it's like in 3D system



3) Quadratic surface (hyperboloid of one sheet) :this form used in the construction of gasoline towers and nuclear power plants in the form of hyperbola and through.

its general equation → $\frac{z^2}{A^2}+\frac{y^2}{B^2}-\frac{x^2}{c^2}=1$ the parallel axis is Z either at level XY its hyperbola while at the Z level of distributed ellipse

4) Quadratic surface (elliptic cone) :

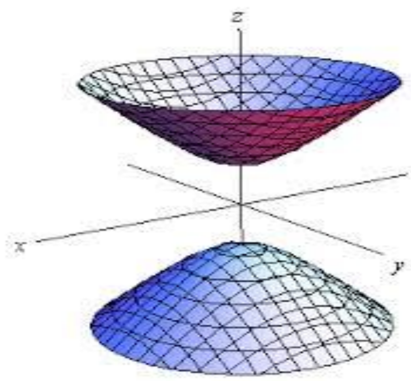


This shape is triangular surface described by an equation whose effects include deleting marks an interrupting lines .the form resulted from some changes in the equation of quadratic surface , so we got the elliptic cone

The general equation is → $\frac{x^2}{A^2}+\frac{y^2}{B^2}-\frac{z^2}{c^2}=0$ It's a Z level of ellipse while at the xz level it's a pair of lines intersecting with the point of origin

also present at the yz level.

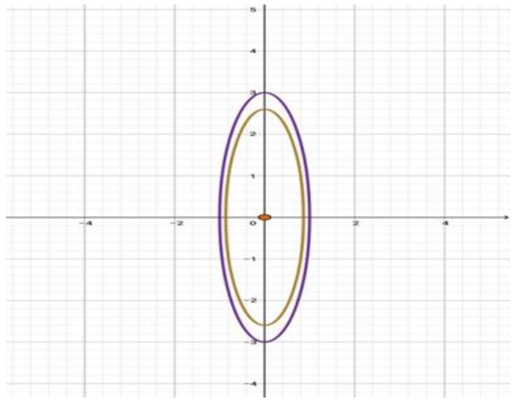
5) Quadratic surface (hyperboloid of two sheets) :



The shape is largely teo pieces of parabola facing each other , which is complex surface this because it comes from two pieces , a the vertical sections are present , and they are oversaturated . it is three-dimesional surface described by an equation that inclides the effects of this surfaces signs of deletion and excess segmentation

Its general equation → $\frac{z^2}{A^2}-\frac{y^2}{B^2}-\frac{x^2}{c^2}=1$

2) THE EQN OF ELLPISOID IS: $\boldsymbol{x^2 + \frac{y^2}{9} + z^2 = 1}$

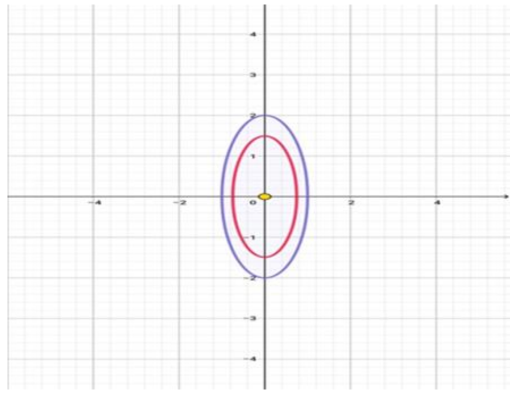


1) Z=K_Z ; K_Z=0,1,2

When kz=0,The equation is → : $x^2 + \frac{y^2}{9} = 1$

When k_z=1,The equation is → :$x^2 + y^2 = \frac{3}{4}$

When k_z=2,The equation is → :$x^2 + \frac{y^2}{9} = 0$

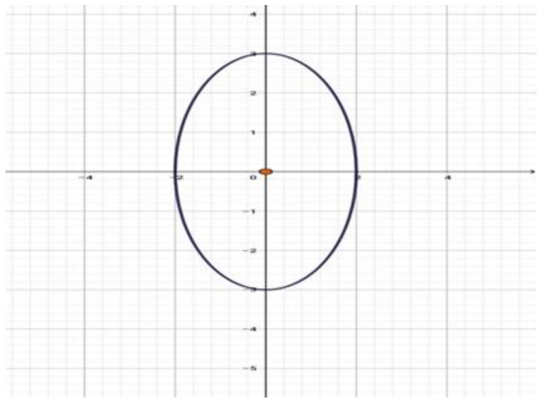


2) Y=K_Y; K_Y=0,2,3

When k_y=0,The equation is: → $x^2 + \frac{z^2}{4} = 1$

When k_y=2,The equation is → :$x^2 + \frac{z^2}{4} = \frac{5}{9}$

When k_y=3,The equation is → :$x^2 + z^2 = 0$



3) X=K_X; K_X=0,1

When k_x=0,The equation is → :$\frac{y^2}{9} + \frac{z^2}{4} = 1$

When k_x=1,The equation is → :$\frac{y^2}{9} + \frac{z^2}{4} = 0$

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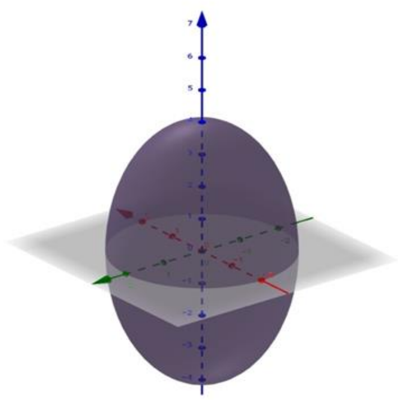
2) F(x, y, z) =$\mathbf{ln(16 - 4x^2 - 4y^2 - z^2)}$

$\boxed{19}$



Domain: $4x^2+4y^2+z^2 <16$

The equation is $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} = 1$

Range:$]-\infty, \ln 16]$. The form is Ellipsoid .It is center (0, 0) and major axis parallel to Z

02 Partial derivative and chain rule

The Partial Derivative

The partial derivative is mathematical function derivation consisting of several variables so that the derivation is for one of these variables ,knowing that the rest of variables are constants and the partial derivative is used in radiation analysis and differential engineering , so it used in a function with several variables and uses the cod (∂) instead of (d) because it's a del derivation in several variables , if del is derived in x,y like f(x , y) for variable x is mathematically expressed F_x or $\frac{df}{dx}$ or $\partial x\, F$ .its function can also be used as three variables F(x , y , z) so that function ,three derivatives and each derivative instead of one of three variables and any compensation in the function results in the tendency of the tangent going in the direction of the its axis .illustration by example : find the first and second partial derivative of the function

$F(x , y)= y^5-3xy$ solution the first derivative is : $F_x= -3y$, $F_y= 5y^4-3x$ the second derivative is $F_{xx}=0$ $F_{yy}=20y^3$

Another Example : find the first partial derivative of function $F(x , y ,z)=xz – 5x^2y^3z^4$ solution $F_x=z-10xy^3z^4$, $F_y=1y^2x^2z^4$, $F_z=20z^3y^3x^2$

Chain Rule

In calculus and the rule of the series there is formula for calculating derived form the functionality compound and this if F and g are functions for difference then the chain rule express a derivative on the compound which assigns the variable x for f(g(x)) in the form of a term and can write the rule of the series in blogging in the following ways the variable z depends on the variable y also depends on the variable x if z and y variables follow it and in this case it is states series on

$$\frac{dz}{dx} = \frac{\partial z}{\partial y} * \frac{\partial y}{\partial z}$$

There are two cases of the basic of the chain rule : 1)**The first case** is : suppose that z =f(x , y) is differentiable function of x and y , where x=g(t) and y=h(t) are both differentiable functions of t .then z is a differentiable function of t and

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x} * \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} * \frac{\partial y}{\partial t}$$

For example : if u $=x^4y + y^2z^3$ where $x = rs\,e^t$, and $z = r^2s \sin t$, and $y= rs^2e^{-t}$ find the value of $\frac{\partial u}{\partial s}$ when r=2 , s=1 , t=0 → Solution: $\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s}$ $=(ax^3y)(re^t)+(x^4+2yz^3)(2rse^t)+(3y^2z^2)(r^2\sin t)$ When r=2 ,s=1and t=0 we have x=2 , y=2 ,z=0 so $\frac{\partial u}{\partial s}$= 64*2+16*4+0*0=192

2)**The second case** of chain rule is : suppose that z=F(x , y) is a differentiable function of x and y , where x=g(s ,t)and y= h(s,t) are differentiable function of s and t then

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$ For example : if $z= e^x \sin y$,where $x= st^2$ and $y= s^2t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ Solution applying case 2 of the chain rule , we got :

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$ $=(t^2\, e^x \sin y)(t^{-2})+(e^x \cos y)(2st)=t^2e^{st^2} \sin(s^2t) +2ste^{st^2} \cos(s^2t)$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$=(e^x \sin y)(2st)+(e^x \cos y)(s^2)= 2ste^{st^2} \sin(s^2 t) + s^2e^{st^2} \cos(s^2 t)$

critical point

When dealing with functions of real variable,the critical point is a value in its field where the gradient is not specified or equal to zero .the application of critical points how we can define max value and min value for example – find the short distance from point (1,0,-2) to the plane x+2y+z=4 solution the distance from any point (x,y,z)to the point (1,0,-2) is d= $\sqrt{(x-1)^2+y^2+(z+2)^2}$ if z=4-x-2y D=$\sqrt{(x-1)^2+y^2+(6-2y-x)^2}$ if d^2=f(x, y)= $(x-1)^2+y^2+(z+2)^2$ by solving the equation $f_x=4x+4y-14=0$ and $f_y=4x+10y-24=0$ the critical point is $(\frac{11}{6},\frac{5}{3})$ by making test $f_{xx}=4$,$f_{yy}=4$,$f_{xy}=10$ D<0 we found that point is min point then by compensation the short distance d =$\frac{5}{6}\sqrt{6}$

2)g (u,v)=f(e^u +sin v, e^u +cos v) evaluate $g_u(0,0)$ and $g_v(0,0)$ by using a table

$g(u,v)=f(x,y)$ $x=e^u+\sin v$ $y=e^u+\cos v$

$x_u=e^u$, $x_v=\cos v$ $y_u=e^u$, $y_v=-\sin v$

$g_u(0,0)=f_x(1,2)*x_u(0,0)+f_y(1,2)*y_u(0,0)$ $g_u(0,0)=3*1+6*1=9$ then $g_u=9$

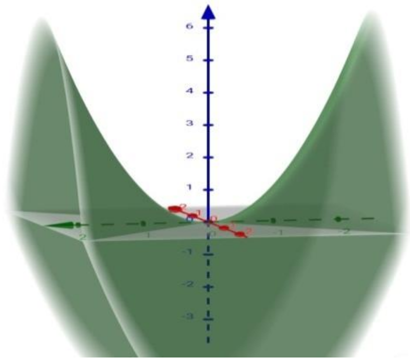
$g_v(0,0)=f_x(1,2)*x_v(0,0)+f_y(1,2)*y_v(0,0)$ $g_v(0,0)=3*1+6*0=3$ then $g_v=3$

$F(x,y)=y^2-x^2$ To find critical point put: $F_x=0$ and $F_y=0$

$F_x=-2x$ $F_x=0$ $-2x=0$ $x=0$

$F_y=2y$ $F_y=0$ $2y=0$ $y=0$

then critical point is (0.0) to classify it we make test

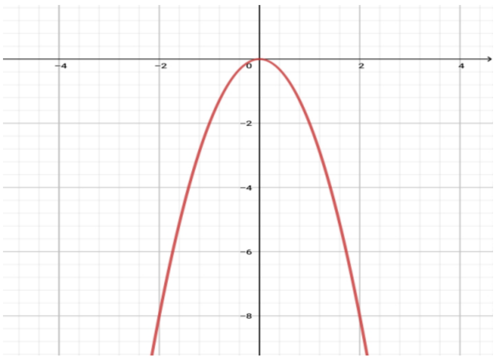


$D=F_{xx}*F_{yy}-(F_{xy})^2$ $F_{xx}=-2$ $F_{yy}=2$ $F_{xy}=0$

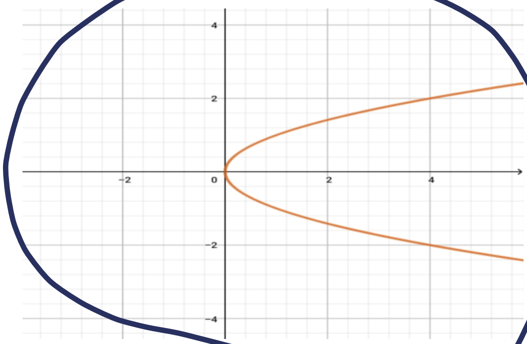
The graph of F(x, y)=y2-x2

$D=(-2*2)-0=-$ $D=-4<0$ $\longrightarrow$ The critical point is saddle point

1)The graph of $F(x,y^*)=-x^2$



2) The graph of $F(x^*,y)=y^2$



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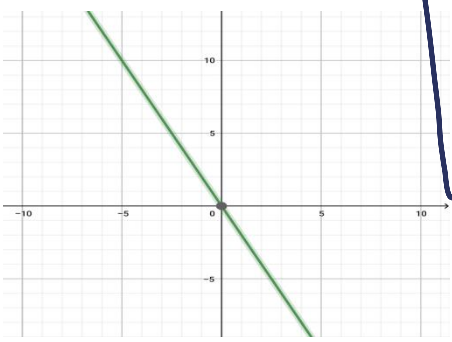
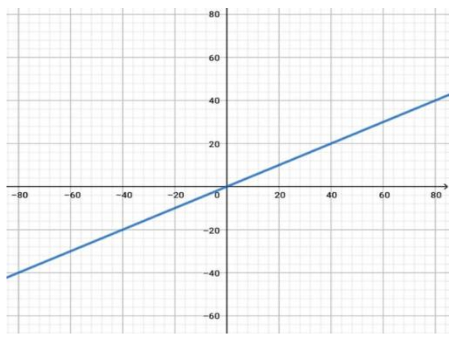
3)The graph of $F_x(x,y^*)=-2x$



4)The graph of $F_y(x^*,y)=2y$



03 Double integral

Double integral

Integration in general is integration that includes knowledge functions that can be in two or more variables where in two variables such as the f (x, y) function is called binary integration while the function contains three variables called the integration of the triangular integration. From there we can explain what binary integration means, but to do so we must understand what the specific integration of a function is in a single variable and positive, it is the area of that area that exists between the x axis and the curve of the function. So bilateral integration, It is a representation of the size of the area that separates the surface that is defined by the function that is in two variables and the content level of its domain. There are two ways to solve bilateral integration. the first way is a fairly easy one and it is a way to use rectangular coordinates and through the formula that comes it can be explained : $\int_c^d \int_a^b f(x,y)dx\,dy$

For example evaluate : $\iint y \sin xy \, dA \quad and \quad 1 \le x \le 2 \quad , \quad 0 \le y \le \pi$

By using rectangular coordinate

$\int_0^\pi \int_1^2 y \sin xy \, dx\, dy = \int_0^\pi -cos2y + \cos y \; dy \quad = \frac{-1}{2} \sin 2y + \sin\; y]^\pi \; = 0$

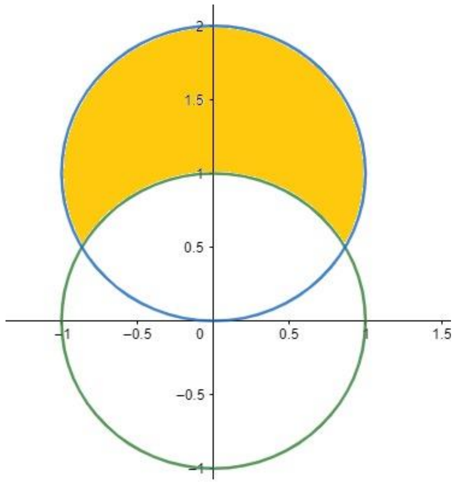
The second method is the use of polar coordinates and the use of that method to facilitate since it was in the form of rectangular coordinates and then became in a polar form which is the division of the circular pieces into subrectangles with parallel sides of the coordinate axes and can understand that use through its formula and through the following example also:

$$\iint xy^2 \, dA \; bounded\; by\; x = 0\,, x = \sqrt{1-y^2}$$

By using polar coordinates $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} x\, y^2 \, dx\, dy \quad = \int_{-1}^1 \frac{1}{2}((1-y^2)y^2) = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}$

It was found that bilateral integration has several different applications, including :surface of area .Average of function, mass of 2D plates, Volume, center of mass and moment of inertia. Double itegeral in volume is V $=\iint (z_2 - z_1)\, dA$ Z_2 ,Z_1 two surfaces D it is the projection of intersection between two surfaces

2-$\iint \frac{x}{x^2+y^2}\, dA$



R is the region inside $x^2+y^2=2y$ and outside $x^2+y^2=1$

$x^2 + y^2 = 1$ $\rightarrow$ The function means equation of circle

It is center (0, 0) and radius =1

$$x^2 + y^2 = 2y \rightarrow x^2 + (y-1)^2 = 1$$

$$\rightarrow this\ function\ means\ equation\ of\ circle$$

$$it\ is\ cinter\ (0,1) and\ radius = 1$$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_{1}^{2sin\theta} cos\theta\ dr\ d\theta \rightarrow \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} r\ cos\theta\ \Big]_{1}^{2sin\theta} d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin\theta\cos\theta - \cos\theta \rightarrow \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin 2\theta - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos\theta$$

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin 2\theta\ d\theta - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} cos\theta \rightarrow -\frac{1}{2}\cos 2\theta\ \Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - sin\theta\Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = 0$$

$$i)\ equition\ of\ the\ plane \rightarrow cx + cy + 6z = 6c \rightarrow z = \frac{6c - cx - cy}{6}$$

$$volume \rightarrow \iint z\ dA \rightarrow \iint \frac{-c}{6}(x + y - 6)\ xdA \ \ = rcos\theta \qquad y = rsin\theta$$

$$\int_0^{2\pi}\int_0^{2} \frac{-c}{6}(rcos\theta + rsin\theta - 6)\ rdr\ d\theta \rightarrow$$

$$\frac{-c}{6}\int_0^{2\pi}\int_0^2 r^2cos\theta + r^2sin\theta - 6r\;dr\;d\theta$$

$$\frac{-c}{6}\int_0^{2\pi}\frac{r^3}{3}cos\theta + \frac{r^3}{3}\sin\theta + 3r^2)_0^2\;d\theta \rightarrow \frac{-c}{6}\int_0^{2\pi}\frac{8}{3}cos\theta + \frac{8}{3}sin\theta - 12\;d\theta$$

$$\frac{-c}{6}\left(\frac{8}{3}sin\theta - \frac{8}{3}cos\theta - 12\theta\right)_0^{2\pi} = 4c\pi$$

$$the\ angle\ between\ the\ planes\ \ \theta = \cos^{-1}\frac{6}{\sqrt{C^2+C^2=36}} = \cos^{-1}\frac{6}{\sqrt{2\left(\frac{V}{4\pi}\right)^2+36}}$$

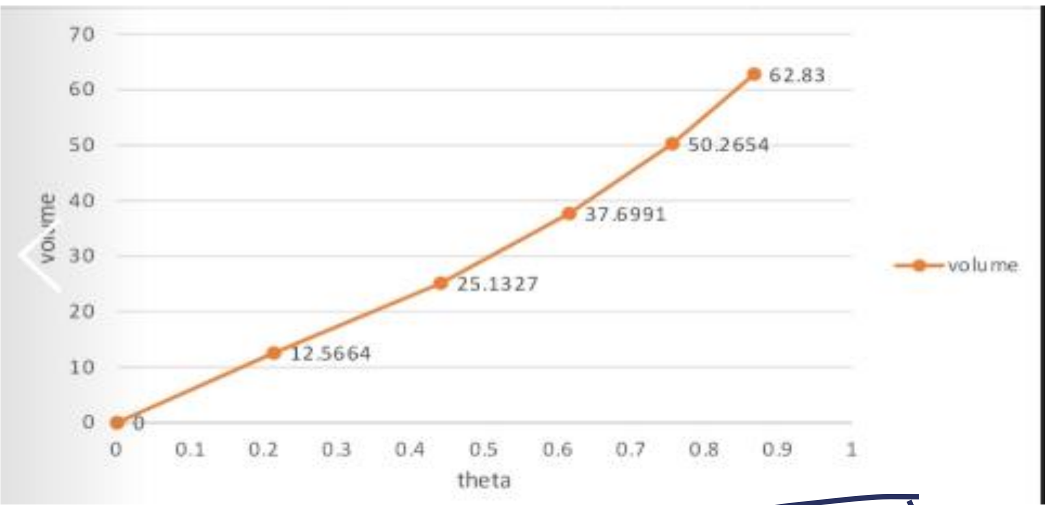
$$substitution\ by \rightarrow c = 0,1,2,3,4,5 \rightarrow get\ this\ point \rightarrow \theta\ in\ x\ axis\ and\ v\ in\ y\ axis$$

$(0,0)$ $(0.2314,4\pi)$ $(0.4405,8\pi)$ $(0.6154,12\pi)$ $(0.7556,16\pi)$ $(0.8671,20\pi)$

$the\ six\ equation\ of\ thw\ plan:$

$18x + 18y + 36z - (36*3) = 0$ $\quad$ $6x + 6y + 36z - 36 = 0$ $\quad$ $12x + 12y + 36z - (36*2) = 0$

$30x + 30y + 36z - (36*5) = 0$ $\quad$ $24x + 24y + 36z - (36*4) = 0$ $\quad$ $Z = 0$



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04 Eigen values and eigen vectors

Eigen values and Eigen vectors

This term is a targeted definition of vectors that convert their digital multiples and the setting to which they are associated. The use of the word Eigen is not understood because that word is derived from German, which is later understood to mean your. Hence, self-values and self-directions are called many terms such as private value and vector, and then use the mathematical term which is self. The linear transformations on which the arrays that influence vectors are built was one of the principles of linear algebra that showed his interest in Transfers. The loss of arrays did not affect vectors, but made self-values, self-directions, and self-spaces of their properties. How do you calculate the self-values and self-vectors that this is done through data that gives you that data is about the matrix that you use to analyze or disassemble the matrix. It was noted that the general matrix has an impact on its values and direction of the vector but also found that the matrix can affect only one of them without changing the other in the sense that it can change the values of the vector and make its direction constant i.e. does not change and vice versa to the following generic formula that is used to create self-values and self-vectors can be explained :

$$\det(A - \lambda I) = 0.$$

assume that you have a square matrix so that its vector is not equal to zero or you can't calculate the self-values and self-vectors and then x is self-destined for the A matrix and if there is a number which is called the self-value of the matrix corresponds to the x vector. Self-values and self-directions are known to have a course other than mathematics, which is one of its applications, which is equivalent to Schrodinger, which is used in physics in addition to having applications in applied mathematics such as finance and also to quantum mechanics.

$$\det\begin{pmatrix} 1-\lambda & 0 & 1 \\ -1 & -1-\lambda & -1 \\ -1 & 0 & -2-\lambda \end{pmatrix} = 0 \rightarrow and\ from\ the\ second\ colume$$

$$\rightarrow -0 + (-1-\lambda)\begin{vmatrix} 1-\lambda & 2 \\ -1 & -2-\lambda \end{vmatrix} - 0 = 0$$

$$\therefore -(1+\lambda)(-2-\lambda+2\lambda+\lambda^2+2) = -(1+\lambda)(\lambda+\lambda^2) = 0$$

$$\therefore -\lambda^3 - 2\lambda^2 - \lambda = 0$$

$$\therefore \lambda = \ -1\ , -1\ , 0$$

$for\ \lambda = -1 \quad and\ dy\ substituting\ in\ the\ characteristic\ equation$

$$\begin{pmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{pmatrix} = 0$$

$$2x_1 + 0x_2 + 2x_3 = 0 \qquad (1)$$

$$-x + 0x_2 - x_3 = 0$$

$$from\ 1\ let\ x_1 = a\ , x_2 = b \qquad \therefore x_3 = -a$$

$$\therefore \quad x = \begin{pmatrix} a \\ b \\ -a \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$for\ \lambda = 0 \quad and\ by\ substituting\ in\ the\ characteristic\ equation$

$$\begin{pmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ 1 & 0 & -2 \end{pmatrix} = 0$$

$$x_1 + 0x_2 + 2x_3 = 0$$

$$-x_1 - x_2 - x_3 = 0$$

$$x_1 : x_2 : x_3 = \begin{vmatrix} 0 & 2 \\ -1 & -1 \end{vmatrix} : \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} : \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} = 2 : -1 : -1$$

$$x = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

$$by\ applying\ the\ diagonalization\ equation \quad A^n = P.D^n.P^{-1}$$

$$A^{2020} = P.D^{2020}.p^{-1}$$

$$P = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix} \qquad D^{2020} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P^{-1} = \frac{1}{|p|} . p^{adj} \qquad coff(p) = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}$$

$$P^{adj} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} \qquad |p| = 1$$

$$\therefore P^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A^{2020} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & -2 \\ -1 & -1 & 0 \end{pmatrix} . \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

$\boxed{30}$

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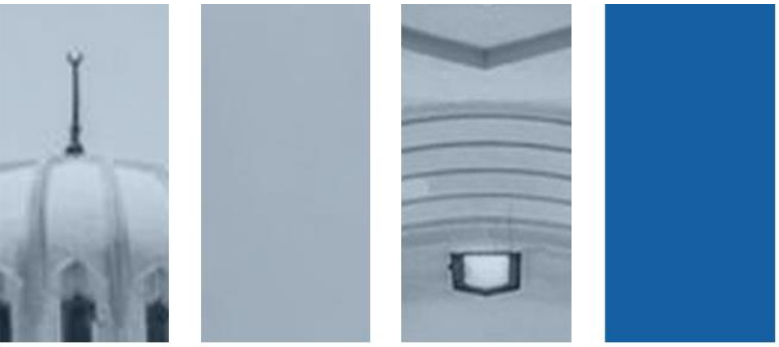

RESEARCH & PROJECT

Program: FRESHMEN

Course Code: PHM 013s

Course Name: Mathematics (2)

Examination Committee

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Spring Semester – 2020

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Submission Contents

01: **Quadratic Surfaces**
02: **Partial Derivative**
03: **Double Integration**
04: **Eigen values and Eigen Vectors**

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01

First Topic

Quadratic Surfaces

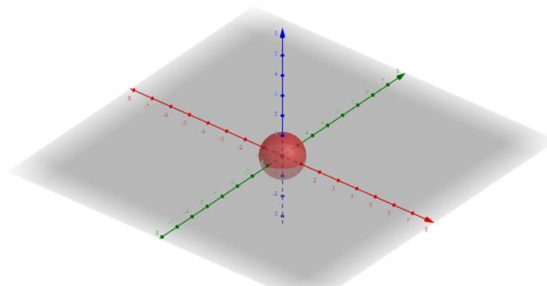
1.1 Discuss different types in details of Quadratic Surfaces given by the equation:

$$Ax^2 + By^2 + Cz^2 + Dz = E$$

1-If A=1, B=1, C=1, D=0, E=1 So,the equation will be

$$x^2 + y^2 + z^2 = 1$$

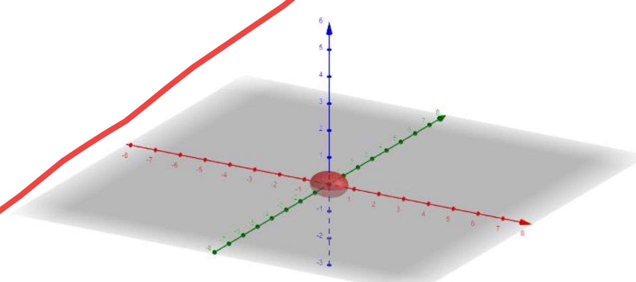
∴ The Quadratic surface will be **sphere**



2-If A ≠ 0, B ≠ A ≠ 0, C ≠ B ≠ A ≠ 0, D= 0, E= 1 So the equation will be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

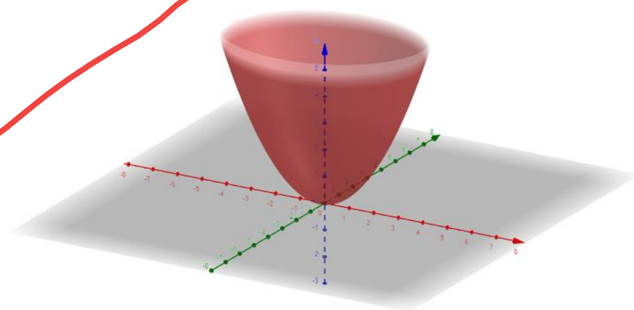
∴ The quadratic surface will be **Ellipsoid**



3-If $A \neq B \neq D \neq 0$,E = C = 0 So,The equation will be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z}{c}=0$$

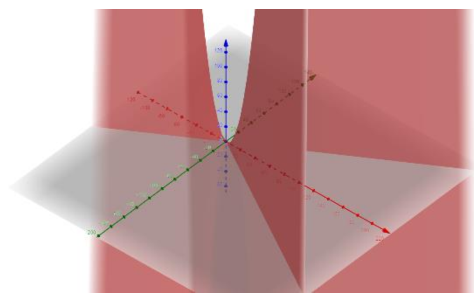
∴ The quadratic surface will be **Elliptic Paraboloid**



4-If $A \neq B \neq D \neq 0$,E =C =0 So,equation will be

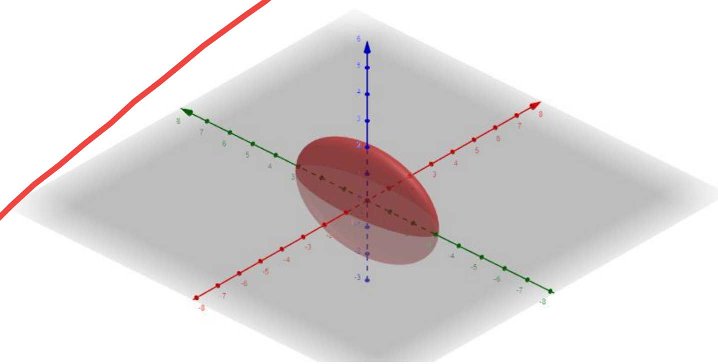
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z}{c}=0$$

∴ The quadratic surface will be **Hyperbolic Paraboloid**

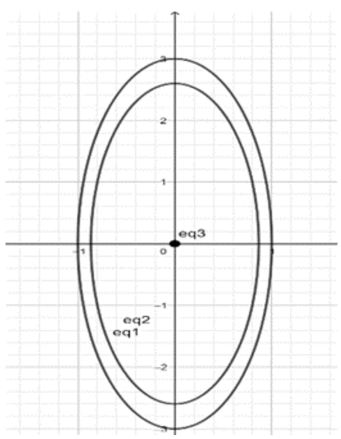


1.2 For the Ellipsoid shown in Figure and given by equation:

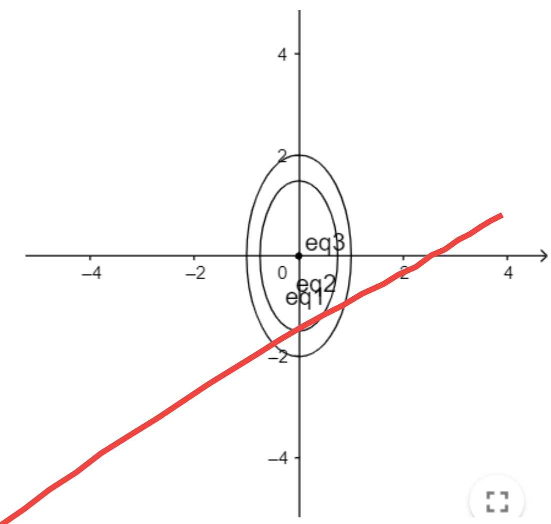
$$x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$$



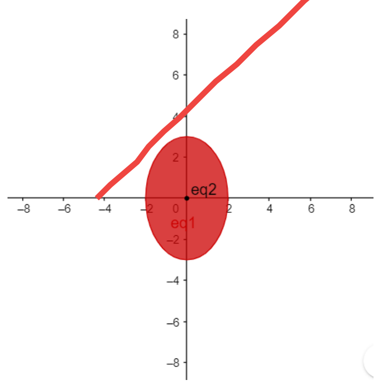
1.2.1 When Z = 0,1,2



1.2.2 When Y = 0,2,3



1.2.3 When X = 0,1



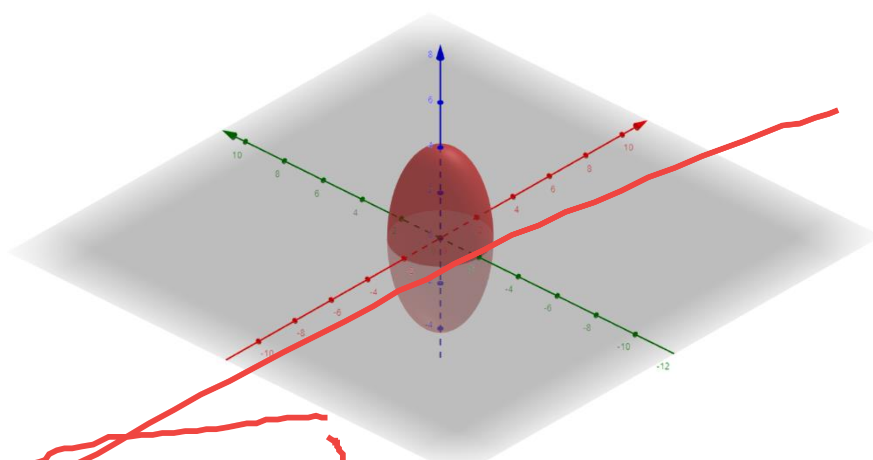
1.3 Find and Sketch the Domain of the function and find its Range: $F(x, y, z) = ln(16 - 4x^2 - 4y^2 - z^2)$

Get Domain by : $16 - 4x^2 - 4y^2 - z^2 > \mathbf{0}$

$\therefore \mathbf{4x^2 + 4y^2 + z^2 = 16}$ $\qquad$ **$(\div$ 16)**

$\therefore \ \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} = \mathbf{1}$ $\rightarrow$ **equation of Ellipsoid whose center $(\mathbf{0}, \mathbf{0}, \mathbf{0})$ and major axis ∥ z-axis**

So the domain is all points which lies inside the Ellipsoid and The **range** is $(-\infty, \ln(16))$



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02

First Topic

The Partial Derivatives

2.1 Discuss the Partial Derivatives and Chain Rule in two different cases and explain the use of Partial Differentiation in obtaining critical points of the surfaces with applications.

2.1.1)**Partial Derivatives** of a function of many unknown variables are their derivatives with respect to one of those variables, with the others stay constant (as versus to the total derivative, in which all variables are allowed to modify). Partial derivatives are used in vector calculus methods and differential geometric applications. The partial derivative of a function which has several variables is considered its derivative with respect to one of these variables .If f is a function of two variables, its partial derivatives are the functions fx and . When we take a general point to calculate the value, it is written as z = F (x, y) where z is a variable which is dependent and (x , y) are variables but are not dependent . For example , v = π $r2$h , here volume (v) depends on radius (r) of a cylinder and height (h) , then the partial derivative of v is fr and fh. If we want to get fx, regard as a constant and differentiate with respect to x.

2.1.2) **The Chain Rule** profess that the derivatives of any composite functions such as $f\big(h(x)\big) = f'\big(h(x)\big).h'(x)$,so it is a way to calculate the derivative of a composite function directly in easy way and it is used in partial derivative as following example:

If $u = x^2y + y^2z^3$, if where $x = f(r,t,s), y = h(r,t,s), z = m(r,t,s)$,So the **Chain Rule** will be:

$$\frac{\partial u}{\partial s} = \frac{\partial u \partial x}{\partial x \partial s} + \frac{\partial u \partial y}{\partial y \partial s} + \frac{\partial u \partial z}{\partial z \partial s}$$

And this equation formed to get the partial derivative of u with respect to s.

Case 1 of the chain rule :

If u = g (x) and v = h (x) where g , h are differentiable function of x , z = f (u , v) where f is a differentiable function of u , v , it means z is a function of x but indirectly way , therefore z=f(g(x) , h(x)) where f is a differentiable function of x , then

$\frac{dz}{} = \frac{\partial z}{}. \frac{du}{} + \frac{\partial z}{}. \frac{dv}{}$ another example , Suppose that is a differentiable function of and ,

$dt \quad \partial u \quad dt \quad \partial v \; dt$

where and are both differentiable functions of .

2.1.2)Case 2 of the chain rule :

If u = g (x , y) and v = h (x , y) , where h , g are differentiable function of x and y , z = f (u , v) , where f is a differentiable function of v , u , therefore z = f (g (x , y) , h (x , y)) , where f is a differentiable function of x , y , then , $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} . \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} . \frac{\partial v}{\partial x}$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} . \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} . \frac{\partial v}{\partial y}$$

Case 2 of the Chain Rule contains three types of variables : x and y are independent variables , u and v are called intermediate variables and z is the dependent variable . In any cases , Suppose that u is a differentiable function of the n variables $(y_1, y_2, y_3, \ldots\ldots, y_n)$ and each of them is a differentiable function of m variables $(x, x, x, \ldots\ldots, x)$, then , $\partial z = \partial z \; . \; \partial y_1 + \partial z \; . \; \partial y_2 + \ldots\ldots + \frac{\partial z}{} . \partial y_n$

2.2 suppose f is differentiable function of x and y and $\boldsymbol{g(u,v) = f(e^u + \sin v\,, e^u + \cos v)}$

By using the values in the table

Let $x = e^u + \sin v$ and $y = e^u + \cos v$,So

$g_u = f_u = x_u . f_x(e^u + \sin v\,, e^u + \cos v) + y_u . f_y(e^u + \sin v\,, e^u + \cos v)$

$g_u = f_u = (e^u). f_x(e^u + \sin v\,, e^u + \cos v) + (e^u). f_y(e^u + \sin v\,, e^u + \cos v)$

By substitution: $g_u(0,0) = 1. f_x(1,2) + 1. f_y(1,2)$ and by using values in table:

$f_x(1,2) = 3\ and\ f_y(1,2) = 6$

$\therefore \boldsymbol{g_u(0,0) = 3 + 6 = 9}$ **(First required)**

$g_v = f_v = x_v . f_x(e^u + \sin v\,, e^u + \cos v) + y_v . f_y(e^u + \sin v\,, e^u + \cos v)$

$g_u = f_u = (\cos v). f_x(e^u + \sin v\,, e^u + \cos v) + (-\sin v). f_y(e^u + \sin v\,, e^u + \cos v)$

By substitution: $g_v(0,0) = 1. f_x(1,2) + 0. f_y(1,2)$ and by using values in table:

$f_x(1,2) = 3\ and\ f_y(1,2) = 6$

$\therefore \boldsymbol{g_v(0,0) = 3 + 0 = 3}$ **(Second required)**

2.3 Find and classify the critical point of surface:

$f(x,y) = y^2 - x^2$, Check the critical point (x^*, y^*) graphically

To find critical point we must put $f_x = 0, f_y = 0$, So:

$f_x = -2x \quad put\ f_x = 0 \quad \rightarrow x = 0$

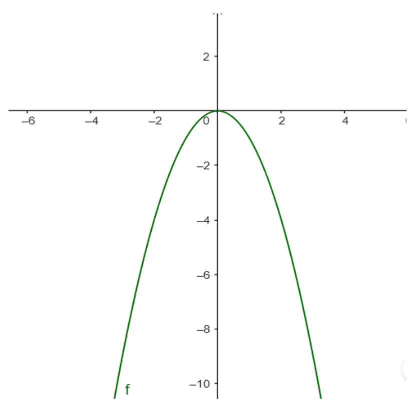
$f_y = 2y \quad put\ f_y = 0 \quad \rightarrow y = 0$

$\therefore$ There is one critical point which is (0 , 0)

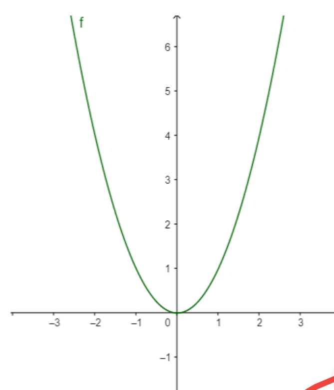
We can get second order derivative: $f_{xx} = -2, f_{yy} = 2, f_{xy} = 0$

$D = f_{xx} f_{yy} - (f_{xy})^2 = (-2)(2) - 0 = -4$

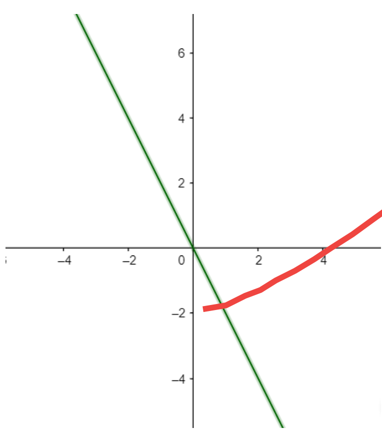
$f(x, y^*)$:



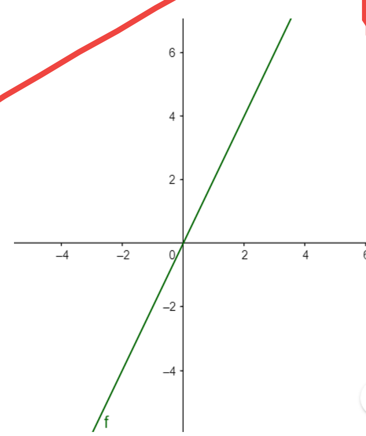
$f(x^*, y)$:



$f_x(x, y^*)$:



$f_y(x^*, y)$:



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03

First Topic

Double Integration

3.1 Discuss the Double Integrals in different coordinate systems.

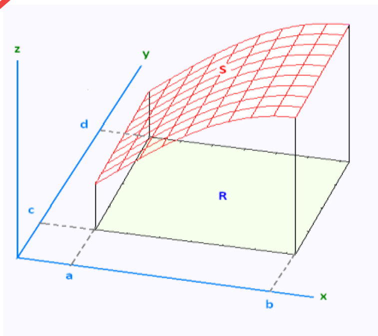
For rectangular coordinate system:

1)3.1.1 Double integral :

we use double integral to find the volume of a solid , $\Delta v = f(x, y)\Delta A$, v =

$\lim_{\Delta x \& \Delta y \to 0} \sum_{\Delta y} \sum_{\Delta x} (x, y)\Delta x \, \Delta y = \iint_A f(x, y)dA$

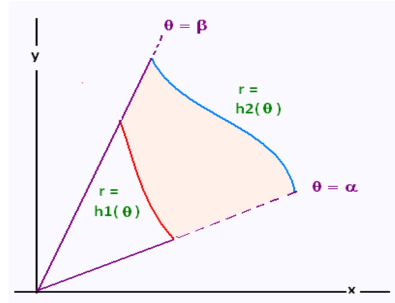
1)Double Integral over rectangular area :



Assumed that f is continuous function $f(x, y) \geqslant 0$ of two variables over rectangular (R) , $R = [c, d] \times [a, b] = \{(x, y) \subset R^2 | a \leqslant x \leqslant b, c \leqslant y \leqslant d|\}$. Function's graph is considered a surface that has equation z = f (x , y) where z is the highest at the point (x , y) of the surface where above the xy plane or above R as the base of the solid is R in xy plane and below the function's graph . Then to find the volume , The first step is to divide the rectangle into subrectangles , each of them has area $\Delta A = \Delta x \, \Delta y$ and sides Δy and Δx , that is happened when divide the interval $[a , b]$ into n subintervals , then $\Delta x = \frac{b-a}{n}$ and divide $[c , d]$ into m subintervals , then $\Delta y = \frac{d-c}{m}$, then $f(x^*_{ij}, y^*_{ij})\Delta A$ is a volume of subrectangles above R_{ij} where $f(x^*_{ij}, y^*_{ij})$ is height of subrectangles and Δx is area of subrectangle . By using this idea for all small rectangles , we obtain the volume of s , $v = \sum_{i=1}^{n}\sum_{j=1}^{m} f(x^*_{ij}, y^*_{ij})\Delta A =$

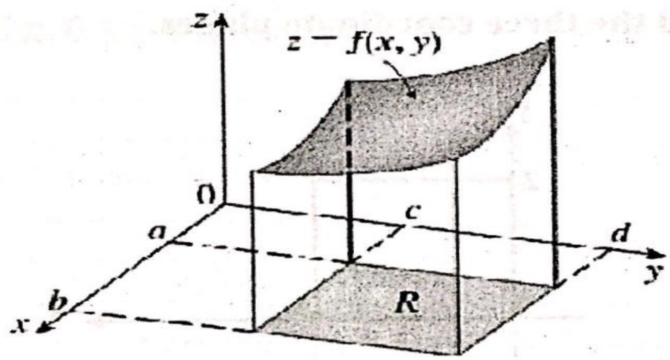
$\lim_{\Delta x, \Delta y \to 0} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x^*_{ij}, y^*_{ij})\Delta A$, we now are ready to use double integral as the sum is a limit and the limit is the volume of the solid (s) , $v = \iint_R f(x, y)dA = \int_c^d \int_a^b f(x, y)dxdy$.

3.1.2) Double integral in polar coordinates :



In polar coordinates , y = rsin θ, x = rcos θ, x^2+y^2=1 then v= $\iint_A f(x,y)dA$ where R is a polar rectangular ,

,By dividing interval $[a , b]$ into n subintervals , then Δr = $\frac{b-a}{n}$ and dividing interval $[\alpha, \ ]$ into m subintervals , then $\Delta\theta = \frac{\beta-\alpha}{m}$, then dividing the polar region into small polar regions (rectangles) , $\iint f(x,y)dA = \int_{\alpha}^{\beta}\int_{r_1(\theta)}^{r_2(\theta)} f(\text{rcos}\,\theta, \text{rsin}\,\theta)\text{r}\, d\, r d\theta$.



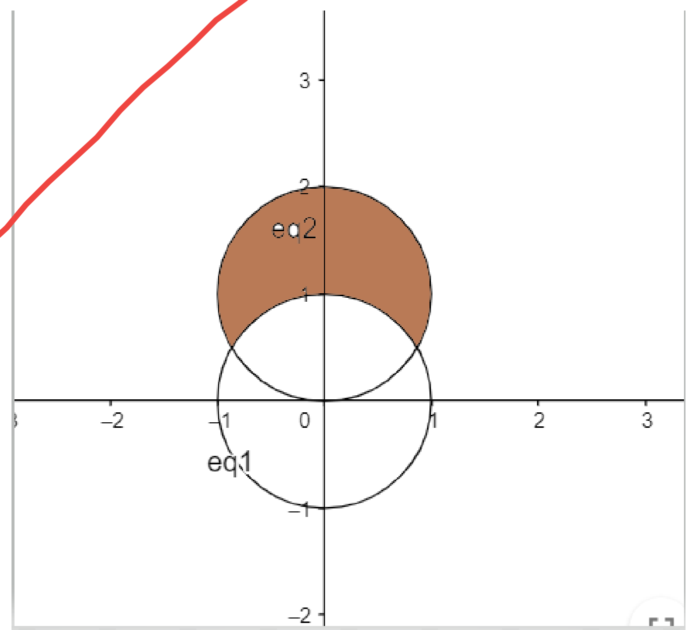
$$A(x) = \int_c^d f(x,y)\,dy$$

$$V = \int_a^b \left[\int_c^d f(x,y)\ dy \right] dx$$

$$= \int_a^b\int_c^d f(x,y)\,dydx = \int_c^d\int_a^b f(x,y)\,dxdy.$$

3.2 Evaluate $\iint_R \frac{x}{x^2+y^2}\, dA$, where R is the region inside $x^2+y^2=2y$ and outside $x^2+y^2=1$

To evaluate $\iint_R \frac{x}{x^2+y^2}\, dA$, we must first sketch the region inside the previous equations



By solving the two equations with each other we found that the two shapes intersects at $(\pm\frac{\sqrt{3}}{2}, \frac{1}{2})$

Also we can get $\tan\theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\therefore\ \theta = 30°$, So it changes from $\frac{\pi}{6}$ *to* $\frac{5\pi}{6}$

$\therefore$ The integration will be:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_1^{2\sin\theta} \frac{r\cos\theta}{r^2} r\, dr d\theta$$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_1^{2\sin\theta} \frac{\cos\theta}{1}\, dr d\theta$$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos\theta\, r\,]_1^{2\sin\theta} d\theta \to\to\to \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} -\cos\theta + 2\sin\theta\cos\theta\, d\theta = 0$$ (by using calc.)

1

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04

First Topic

Eigen values and Eigen vectors

Discuss the determination of the Eigen values and Eigen vectors for matrix A with applications in determining power of the matrix. Then solve the following problem:

For matrix A = $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-2} \end{pmatrix}$

If A is a square matrix, then the matrix A-XI is said to be the **characteristic matrix of A.** And in Algebra, **Eigen vectors** or characteristic vectors is a non-zero vector which vary with a scalar factor and this scalar corresponding factor is called **Eigen value** by which **Eigen vector** is scaled.

And the applications of it is to determine High power of matrix such as A^{2020}.

From the characteristic root equation of a matrix:

$$\boldsymbol{det}\begin{pmatrix} \mathbf{1-\lambda} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{-1-\lambda} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-2-\lambda} \end{pmatrix} = \mathbf{0} \qquad \text{(compute determinant from the second column)}$$

$$\mathbf{0} + (\mathbf{-1-\lambda})\begin{vmatrix} \mathbf{1-\lambda} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-2-\lambda} \end{vmatrix} = \mathbf{0}$$

$$\therefore (-1-\lambda)(-2-\lambda+2\lambda+\lambda^2+2) = -(1+\lambda)(\lambda+\lambda^2) = 0$$

$$\therefore\ -\lambda^3 - 2\lambda^2 - \lambda = 0$$

$$\therefore\ \lambda = -1,0$$

By subs. in the characteristic equ. $\lambda = -1$

$$\begin{pmatrix} \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-1} \end{pmatrix} = \mathbf{0}$$

$$2x_1 + 0x_2 + 2x_3 = 0 \qquad \rightarrow \qquad (1)$$

Let $x_1 = a\ ,\ \ x_2 = b\ \ ,\ \ x_3 = -a$

$$\therefore x = \begin{pmatrix} a \\ b \\ -a \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

For $= 0$:

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} & \mathbf{-2} \end{pmatrix} = \mathbf{0}$$

$x_1 + 0x_2 + 2x_3 = 0$

$x_1 : x_2 : x_3 = 2 : -1 : -1$

$$\therefore x = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

By applying the diagonalization equation $\quad A^n = T.B^n.T^{-1}$

$\therefore A^{2020} = T.B^{2020}.T^{-1}$

$$T = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$B^{2020} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^{-1} = \frac{1}{|T|}.T_{adj}$$

$$T_{adj} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$$ (by using calc.)

$|T| = 1 \neq 0$

$$T^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A^{2020} = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}.\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.\begin{pmatrix} 1 & 0 & 1 \\ -1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

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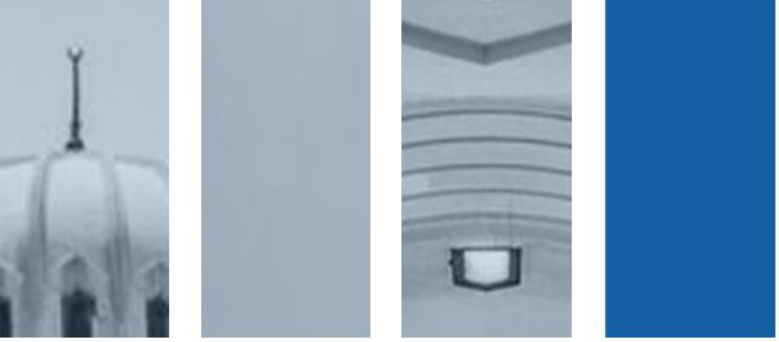
Reference used:

1. Calculus 6th-edition James Stewart
2. Elementary linear algebra with applications 9th edition



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RESEARCH & PROJECT SUBMISSIONS

Program: Sophomore

Course Code: ***PHM013s***

Course Name: ***Mathematics (2)***

Examination Committee

Prof. Niveen Badra

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Spring Semester – 2020

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Submission Contents

01: **A quadratic surfaces, traces of the Ellipsoid, Partial Derivatives and Chain Rules**
02: **The Double Integrals**
03: **The Triple Integrals**
04: **Linear System of Equations**

+ $\boxed{7}$

1) i) Discuss different types in details of Quadratic Surfaces given by the equation:

$$A\ x^2 + B\ y^2 + C\ z^2 + D\ z = E \qquad (8\ \%)$$

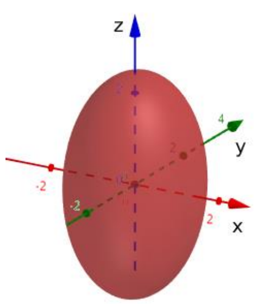
i) A quadratic surfaces are the graphs in space of a second degree equation in x, y and z. The equation quadratic surfaces A x^2 + B y^2 + C z^2 + D Z = E, where A, B, C, D, and E are constants. The basic quadratic surfaces are ellipsoid, Elliptic paraboloid, cone, Hyperboloid. Sphere is special cases of ellipsoid.

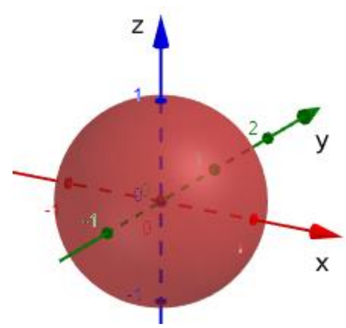
A x^2 + B y^2 + (z^2 +1) z = E ------------ (A)

Now we are considering different types of a quadratic surfaces represented by equation (A)

1) If A, B, C, E are positive and D = 0, then equation (A) represents ellipsoid.

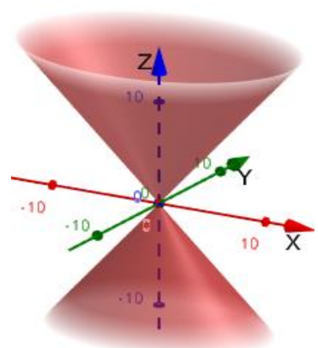
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **Sphere is special case of ellipsoid $x^2 + y^2 + z^2 = a^2$**





2) If C = E = 0 , D < 0 , A and B have the same sign, then equation (A) represents Elliptic paraboloid .

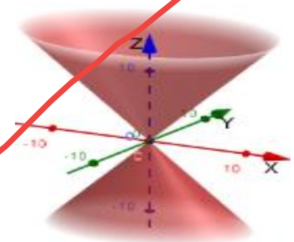
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c}$$



3) If C = E = 0 , D < 0 , A and B have the different signs, then equation (A) represents hyperbolic paraboloid .

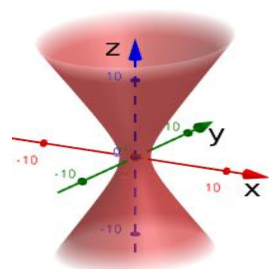
4)) If D = E = 0 , D < 0 , and C is negative, then equation (A) represents Cone.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



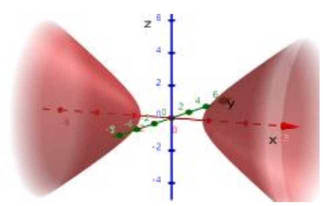
5) If A, B, E are positive , D = 0 , and C is negative, then equation (A) represents Hyperboloid (one sheet).

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

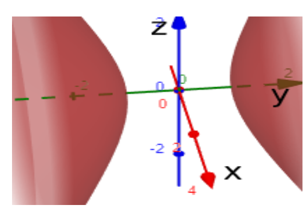


6) If A, B are negative, and C, E are positive and D = 0 , then equation (A) represents Hyperboloid (Two sheets).

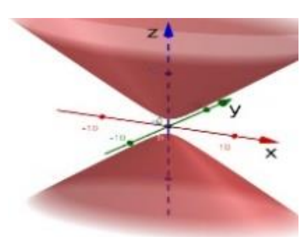
$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ **the axis is parallel to x-axis**



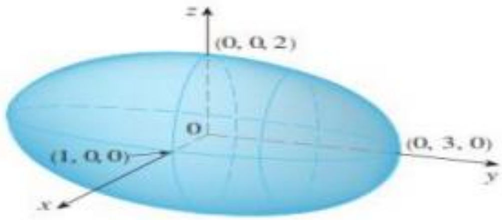
$-\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ **the axis is parallel to y-axis**



$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **the axis is parallel to z-axis**



ii) For the Ellipsoid shown in Figure and given by the equation: $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$



Graph the traces of the ellipsoid (You may use Excel Sheets) when **(6 %)**

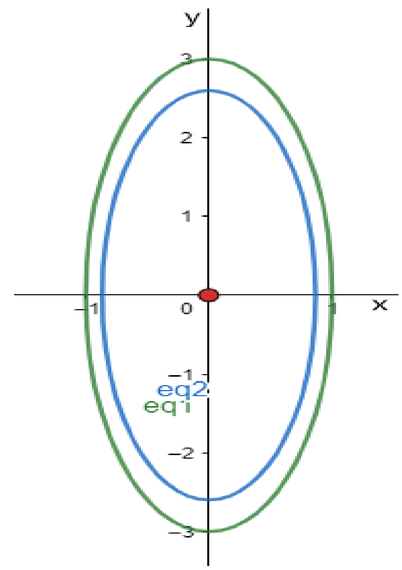
ii) Given Equation of ellipsoid $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

1) Z = K$_Z$

z =0 $x^2 + \frac{y^2}{9} + \frac{0}{4} = 1$ $x^2 + \frac{y^2}{9} = 1$

z =1 $x^2 + \frac{y^2}{9} + \frac{1}{4} = 1$ $x^2 + \frac{y^2}{9} = \frac{3}{4}$ $\frac{x^2}{\frac{3}{4}} + \frac{y^2}{\frac{27}{4}} = 1$

z =2 $x^2 + \frac{y^2}{9} + \frac{4}{4} = 1$ $x^2 + \frac{y^2}{9} = 0$



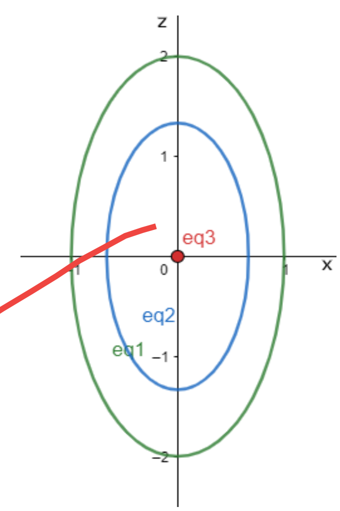
2) Z = K$_Y$

y = 0 $x^2 + \frac{0}{9} + \frac{z^2}{4} = 1$ $x^2 + \frac{z^2}{4} = 1$ --------eq1

y = 2 $x^2 + \frac{4}{9} + \frac{z^2}{4} = 1$ $x^2 + \frac{z^2}{4} = \frac{5}{9}$

$\frac{x^2}{\frac{5}{9}} + \frac{z^2}{4} = \frac{5}{9}$ --------eq2

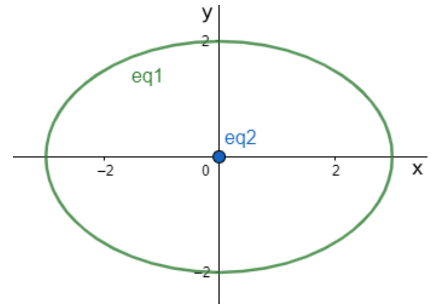
y = 3 $x^2 + \frac{9}{9} + \frac{z^2}{4} = 1$ $x^2 + \frac{z^2}{4} = 0$ --------eq3



3) $X = K_X$

$x = 0 \quad 0 + \frac{y^2}{9} + \frac{z^2}{4} = 1 \qquad \frac{y^2}{9} + \frac{z^2}{4} = 1$ --------eq1

$x = 1 \quad 1 + \frac{y^2}{9} + \frac{z^2}{4} = 1 \qquad \frac{y^2}{9} + \frac{z^2}{4} = 0$ --------eq2



iii) Find and sketch the domain of the following function then find the range
$f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2)$ (6 %)

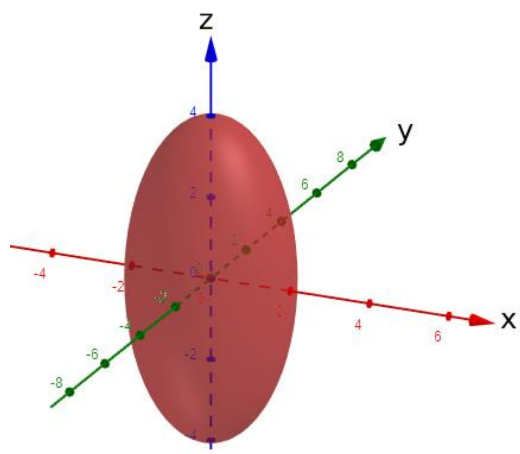
iii) $f(x, y, z) = \ln(16 - 4x^2 - 4y^2 - z^2)$

To find the domain $16 - 4x^2 - 4y^2 - z^2 > 0$ $\quad 16 < 4x^2 + 4y^2 + z^2$

$16 = 4x^2 + 4y^2 + z^2$

Divided by 16 $\quad \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{4} = 1$

This equation expresses an ellipsoid with center (0 , 0 , 0)



The domain is all points that lies inside ellipsoid.

Range:] -∞: ln(16)]

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2) i) Discuss the Partial Derivatives and Chain Rules in different two cases and explain the use of Partial Differentiation in obtaining critical points of the Surfaces with applications. (8 %)

i) a **partial derivative** of a function consists of variables is its derivative with respect to one of those variables, with the others held constant.

Z = F (x , y) it has more than one derivative

the partial derivative of z with respect to x is denoted F_x , $\frac{\partial f}{\partial x}$

he partial derivative of z with respect to x is denoted F_y , $\frac{\partial f}{\partial y}$

$$F_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x + h_1 y) - f(x, y)}{h}$$

$$F_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x_1 y + h_1) - f(x, y)}{h}$$

First order partial derivatives:

$$\frac{\partial f}{\partial x} = f_x = \partial_x f.$$

Second order partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = \partial_{xx} f = \partial_x^2 f.$$

Chain Rules

The chain rule (case 1) z = f (x , y) is a differentiable function of x and y , where x = g (t) and y = h (t) are both differentiable functions of t and z is a differentiable function of t and $\frac{d_z}{dt} = \frac{\partial f}{\partial x}\frac{d x}{d t} + \frac{\partial f}{\partial y}\frac{d y}{d t}$

The chain rule (case 2) z = f (x , y) is a differentiable function Of x and y, where x = g (r , s) and x = h (r , s) are differentiable functions of *r* and *t*

$$\frac{d_z}{dr} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} \qquad \frac{d_z}{ds} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

Case 2 of the Chain Rule contains three types of variables: z is the dependent variables, r and s are independent variables.

Partial derivatives are used to determine critical points for a function of two variables.

The critical point of the multivariable function is a point where the point derivative of the first order of this function = zero.

Apply a second derivative test in order to identify a critical point as a local maximum, local minimum, and saddle point for a function of two variables. checkup critical points to find absolute maximum and minimum value for a function of two variables.

One of the most useful applications for derivatives of the function of one variable is the determination of maximum or minimum values. This application is also important for the functions of two or more variables. The main ideas of finding critical points and using derivative tests are still valid, but new wrinkles appear when assessing the results.

ii) Suppose f is differentiable function of x and y and
$g(u, v) = f(e^u + \sin v, e^u + \cos v)$
Use the following table of values to evaluate $g_u(0,0)$ and $g_v(0,0)$. (6 %)

	f	g	f_x	f_y
(0,0)	6	3	8	4
(1,2)	2	5	3	6

ii) $g(u,v) = f(e^u + \sin v, e^u + \cos v)$
let $x = e^u + \sin v$ and $y = e^u + \cos v$
Now $g(u,v) = f(x,y)$

By using chain rule $\frac{dg}{du} = \frac{df}{dx}\frac{dx}{du} + \frac{df}{dy}\frac{dy}{du}$

$g_u = f_u = f_x(e^u + \sin v, e^u + \cos v)\, x_u + f_y(e^u + \sin v, e^u + \cos v)\, y_u$

$g_u = f_x(e^u + \sin v, e^u + \cos v)\, e^u + f_y(e^u + \sin v, e^u + \cos v)\,(e^u)$

$g_u(0,0) = f_x(1,2) + f_y(1,2)$

$f_x(1,2) = 3$ and $f_y(1,2) = 6$ $\quad g_u(0,0) = 3 + 6 = 9$

$\frac{dg}{dv} = \frac{df}{dx}\frac{dx}{dv} + \frac{df}{dy}\frac{dy}{dv}$

$g_v = f_v = f_x(e^u + \sin v, e^u + \cos v)x_v + f_y(e^u + \sin v, e^u + \cos v)y_v$

$g_v = f_x(e^u + \sin v, e^u + \cos v)(\cos v) + f_y(1,2)*0$

$f_x = 3$ $\quad f_y(1,2) = 6$ $\quad g_v(0,0) = 3+0 = 3$

iii) Find and classify the critical point (if any) of the Surface: $f(x, y) = y^2 - x^2$
Check the critical point (x^*, y^*) grahically (You may use Excel Sheets) by graphing: $f(x, y^*)$; $f(x^*, y)$; $f_x(x, y^*)$; $f_y(x^*, y)$ (6 %)

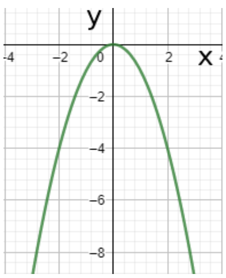
iii) $f(x, y) = y^2 - x^2$ critical points f_x, f_y : $f_x = -2x$ $f_x = 0$ $x = 0$

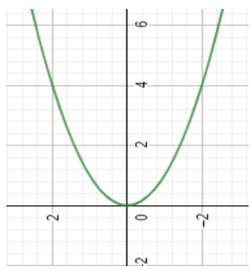
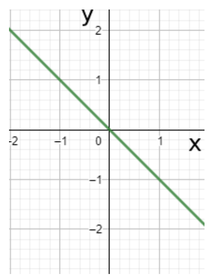
$f_y = 2y$ $f_y = 0$ $y = 0$ critical point (0 , 0)

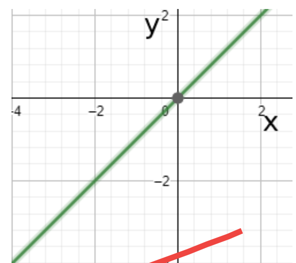
the second order derivative $f_{xx} = -2$

$f_{yy} = 2$ $D = f_{xx} f_{yy} - (f_{xy})^2 = -4 - 0 < 0$

$f_{xy} = 0$ (0 , 0) saddle point





Other solve: $f(x, y) = y^2 - x^2$ $f_x(x, y) = 2x$

$f_y(x, y) = -2y$ $f_{xx}(x, y) = 2y$

$f_{yy}(x, y) = -2$ $f_{xy}(x, y) = 2$

From the definition of critical point, a critical point is a point in the domain of a function where the function is either not differentiable or the derivative is equal to zero.

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$f_x(x, y) = f_y(x, y) = 0$ $2x = 0$ $-2y = 0$

Therefore from two equation we get x = 0 and y = 0 so (0,0) is the saddle point

3) i) Discuss the Double Integrals in different coordinate systems; Rectangular and Polar with applications. (8 %)

i)

Let a single valued and bounded function of $f(x,y)$ of two independent variable x,y is defined in a closed region c of the x-y plane. We divide this region c in n sub regions of areas, $\delta A_1, \delta A_2 \ldots, \delta A_n$.

Let (x_r, y_r) be any point inside elementary area δA_r.

Consider the sum.

$$f(x_1,y_1)\,\delta A_1 + f(x_2,y_2)\,\delta A_2 + \ldots + f(x_n,y_n)\,\delta A_n$$

$$= \sum_{r=1}^{n} f(x_r, y_r) \cdot \delta A_r \quad \text{——} (1)$$

Increase the number of these sub regions indefinitely by taking smaller and smaller elementary areas. The limit of the sum in (1), if it exist, as $n \to \infty$ is called double integeral of $f(x,y)$ over the region C and is donated by

$$\iint_C f(x,y)\, dA$$

So,
$$\iint_C f(x,y)\,dA = \lim_{n \to \infty} \sum_{r=1}^{n} f(x_r, y_r) \cdot \delta A_r$$
$$\delta A_r \to 0$$

Substitution Method For Double Integral:

When we evaluate a double integral it is often found conviniend to make substitutions for change of variables. We substitute

$$x = \phi_1(u,v), \quad y = \phi_2(u,v)$$

then $dx\,dy$ is replaced by $|J|\, du\, dv$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Hence C' is the region in u-v plane

corresponding to the region C of x,y plane

The $|J|$ is called Jacobian of x,y with respect to u,v.

Thus we replace x,y by their values in terms of u,v. and the element of area dx, dy by $|J|\,du\,dv$ and region of integration C by C'.

Application in Polar System:

Assume a region $x^2+y^2 \leq a^2$ and the integral is a function of x^2+y^2, we use the transformation.

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\therefore \quad |J| = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= \pi(\cos^2\theta + \sin^2\theta)$$

$$= \pi(1)$$

$$= \pi$$

If c' is the transformed region of c,

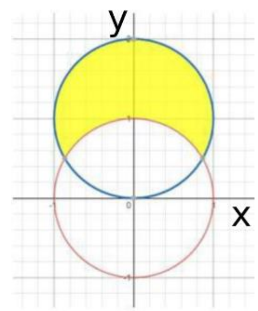
then.

$$\iint_{c} f(x,y)\,dx\,dy = \iint_{c'} f(r\cos\theta, r\sin\theta)\,|J|\,dr\,d\theta$$

$$= \iint_{c'} (f\, r\cos\theta, r\sin\theta)\, r\,dr\,d\theta.$$

ii) Evaluate $\iint_R \frac{x}{x^2+y^2}\,dA$, where R is the region inside $x^2+y^2=2y$ and outside $x^2+y^2=1$ **(6 %)**

II) The region blow the two surfaces can be find by piotting it and then finding limit of x and y. we can find the integral.

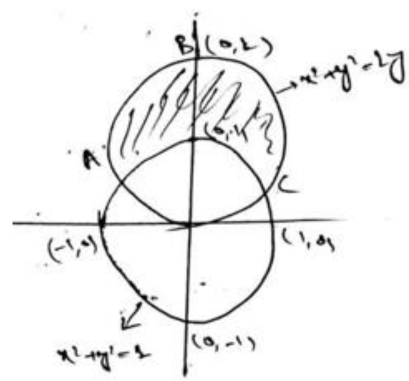


$C1 = x^2 + y^2 = 1$

$C2 = x^2 + y^2 = 2y$

$X^2 + (y-1)^2 = 1$

Region R is colored region so to find the points of intersection A and C.



$x^2 + y^2 = 1$ and $x^2 + y^2 = 2y$ $\quad 2y = 1 \quad y = \frac{1}{2}$

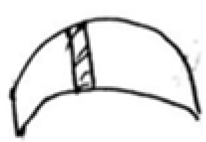
and $x = \pm\sqrt{1-y^2} = \pm\sqrt{\frac{3}{2}}$

$A = (\frac{-\sqrt{3}}{2}, \frac{1}{2})$ and $C = (\frac{\sqrt{3}}{2}, \frac{1}{2})$ $\quad$ B (0 , 2) $\quad$ D (0 , 1)

Vertical strip

$y = \sqrt{1-x^2}$ to $y = 1 + \sqrt{1-x^2}$

$x = \frac{-\sqrt{3}}{2}$ to $x = \frac{\sqrt{3}}{2}$



$$I = \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \int_{\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \frac{x}{x^2+x^2}\, dy\, dx = \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \left(\frac{x}{x}\tan^{-1}(\frac{y}{x})\right)\left(\frac{x}{2}\tan^{-1}(\frac{y}{x})\right)\Big|_{\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} dx$$

$$= \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right) - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) dx$$

$$= \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{\frac{1+\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{x}}{1+\left(\frac{1+\sqrt{1-x^2}}{x}\right)\left(\sqrt{1-x^2}\right)}\right) dx \;=\; \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{x}{1-\sqrt{1-x^2}}\right) dx =$$

So $\quad I = \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{x}{1-\sqrt{1-x^2}}\right) dx$

Put $x = \sin\theta$, $dx = \cos\theta \, d\theta$

$x = \frac{-\sqrt{3}}{2}$, $0 = \frac{-x}{3}$, $x = \frac{\sqrt{3}}{2}$, $0 = \frac{x}{3}$

$$I = \int_{\frac{-x}{3}}^{\frac{-x}{3}} \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right) \cos\theta \, d\theta = \frac{1}{2}\,(4\ \sin\theta + \cos\theta)\Big|_{\frac{-x}{3}}^{\frac{x}{3}} = 0$$

iii) Use Double Integration to Evaluate the Volume (V) bounded by the Cylinder: $x^2 + y^2 = 4$ and the Plane P passing through the points $(6, 0, 0)$; $(0, 6, 0)$ and $(0, 0, c)$ where $0 \le c \le 5$ above xy – plane in terms of c. Then, Sketch the graph between the angle θ between the plane P and the xy – plane and the Volume (V) when $c = 0, 1, 2, 3, 4, 5$

(You may use Excel Sheets). (6 %)

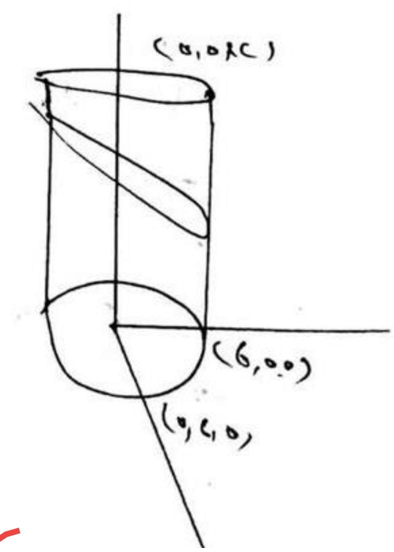
III) To find the volume between the surface f(x , y) and xy plane we should first find equation of the given plane that is f(x , y) as equation $\begin{vmatrix} x-6 & y-0 & z-0 \\ 0-6 & 6-0 & 0-0 \\ 0-6 & 0-0 & c-0 \end{vmatrix} = 0$

C (x - 6) + y + 6z = 0 $\qquad$ z = $\frac{c}{6}(6 - x - y)$ $\qquad$ Now volume

V = $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f\ (x\ , y)\ dy\, dx \quad = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (6 - x - y)\ dy\, dx$

= $\frac{c}{6}\int_{-1}^{1}\left(6y - xy - \frac{y^2}{2}\right)_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$

V = $\frac{c}{6}\int_{-1}^{1} 2\sqrt{1-\ x^2}\ (6 - x)\ \ dx \quad = \quad \frac{c}{6} \times 6x$ = cx



To sketch the graph bw angle and and volume, we should find first angle between x , y plane and given plane using formula of angle between planes

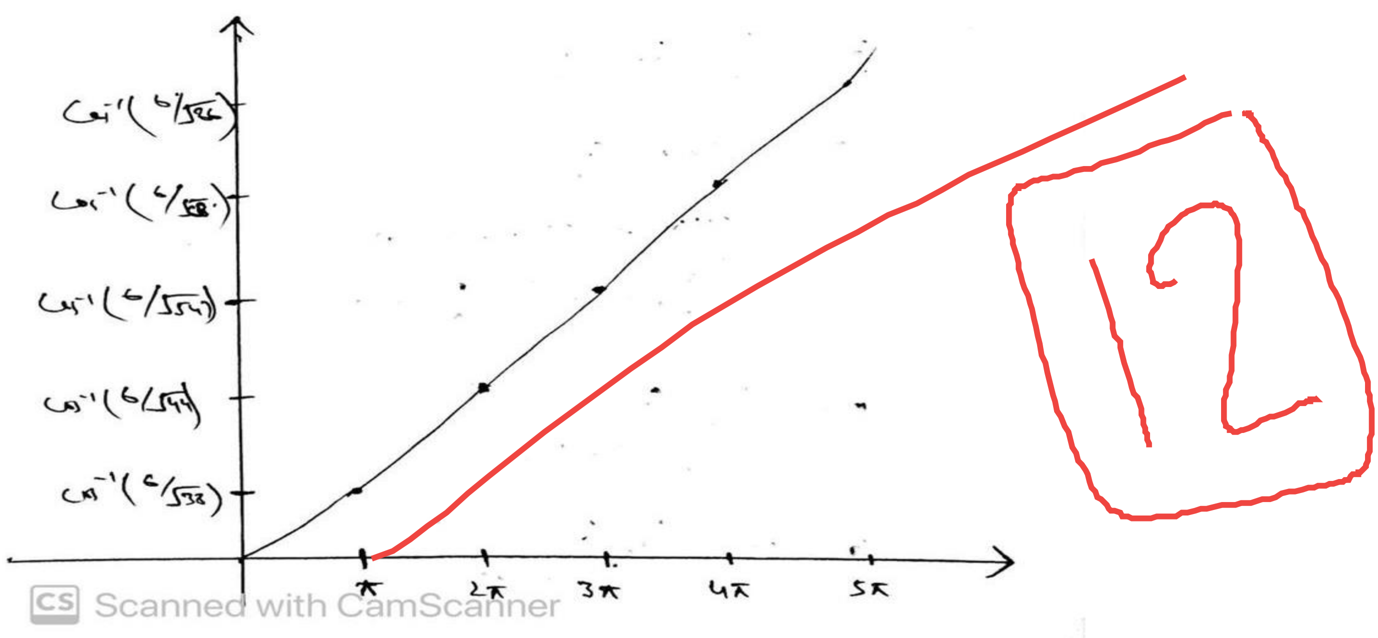
P_1 : z = 0 $\qquad$ $p_2 = \frac{c}{6}\ (6 - x - y)$ $\qquad$ or cx + cy - 6z – 6 = 0

$\cos\theta \ = \left|\frac{(c)(0) + (c)(0) + (c)(1)}{(\sqrt{0^2 + 0^2 + 1^2})(\sqrt{c^2 + c^2 + 6^2})}\right| \ = \ \cos\theta \ = \left|\frac{6}{(\sqrt{2c^2 + 36})}\right|$

$\theta = \cos^{-1}\left|\frac{6}{(\sqrt{2c^2 + 36})}\right|$ $\qquad$ and $\qquad$ v = cx

C	θ	V
0	0	0
1	$\cos^{-1}\frac{6}{\sqrt{38}}$	x
2	$\cos^{-1}\frac{6}{\sqrt{44}}$	2x
3	$\cos^{-1}\frac{6}{\sqrt{54}}$	3x
4	$\cos^{-1}\frac{6}{\sqrt{68}}$	4x
5	$\cos^{-1}\frac{6}{\sqrt{86}}$	5x

From the table



Algebra

1)

$$\begin{pmatrix} -4 & 5 & -2 \\ -3 & 2 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$$

A = LU

$$A = \begin{pmatrix} a & 0 & 0 \\ k & \beta & 0 \\ L & M & Y \end{pmatrix}\begin{pmatrix} 1 & R & S \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}$$

L U

$\propto = -4$ $\propto R = 5$ R $= \frac{-7}{4}$ $\propto$S = -2 S $= \frac{1}{2}$

K = -3 KR + $\beta T = -1$ T $= \frac{-2}{7}$

L = 1 LR + M = 1 M $= \frac{9}{4}$ LS + MT + y =0 y $= \frac{1}{7}$

LU = A

AX = B LUX = B LY = B

$$\begin{pmatrix} -4 & 0 & 0 \\ -3 & \frac{-7}{4} & 0 \\ 1 & \frac{9}{4} & \frac{1}{7} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$$

$Y_1 = \frac{-5}{4}$ $y_2 = \frac{-1}{7}$ $y_1 = 4$

$$\begin{pmatrix} 1 & \frac{-5}{4} & \frac{1}{2} \\ 0 & 1 & \frac{-2}{7} \\ 1 & \frac{9}{4} & \frac{1}{7} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{-5}{4} \\ \frac{-1}{7} \\ 4 \end{pmatrix}$$

$X_1 = -2$ $x_2 = 1$ $x_3 = 4$

2) Discuss the determination of the Eigen values and Eigen vectors for matrix A with applications in determining power of the matrix. Then solve the following problem:

For the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{pmatrix}$, **find** A^{2020} .

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$$= \begin{vmatrix} 1-\lambda & 0 & 2 \\ -1 & -1-\lambda & -1 \\ 1 & 0 & -2-\lambda \end{vmatrix} = -\lambda^3 - 2\lambda^2 - \lambda = -\lambda\left(\lambda^2 + 2\lambda + 1\right)$$

$= \lambda = 0, -1, -1$ $\qquad\qquad$ $\lambda = 0$

$(A - \lambda I) = 0$ $\qquad\qquad$ $x\lambda_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$(A + I)x = 0$ $\qquad\qquad$ $x\lambda_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\qquad\qquad$ $x\lambda_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\qquad$ $p = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\qquad$ $p^{-1} = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$$A^{2020} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}^{2020} \begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad A^{2020} = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Mathematics (2) – PHM 013s Spring 2020

University: Ain Shams Faculty: Engineering

Assessment Methods /ILO Matrix

Final Term Exam	a1	a2	a3	a4	b1	b2	b3	b4	c1	d1
Question 1	●			●	●					
Question 2			●		●		●			
Question 3	●		●				●			
Question 4			●		●		●			
Question 5				●			●			
Question 6	●				●		●			